Nonlinear Low Frequency Wave Phenomena in Space Plasmas

by

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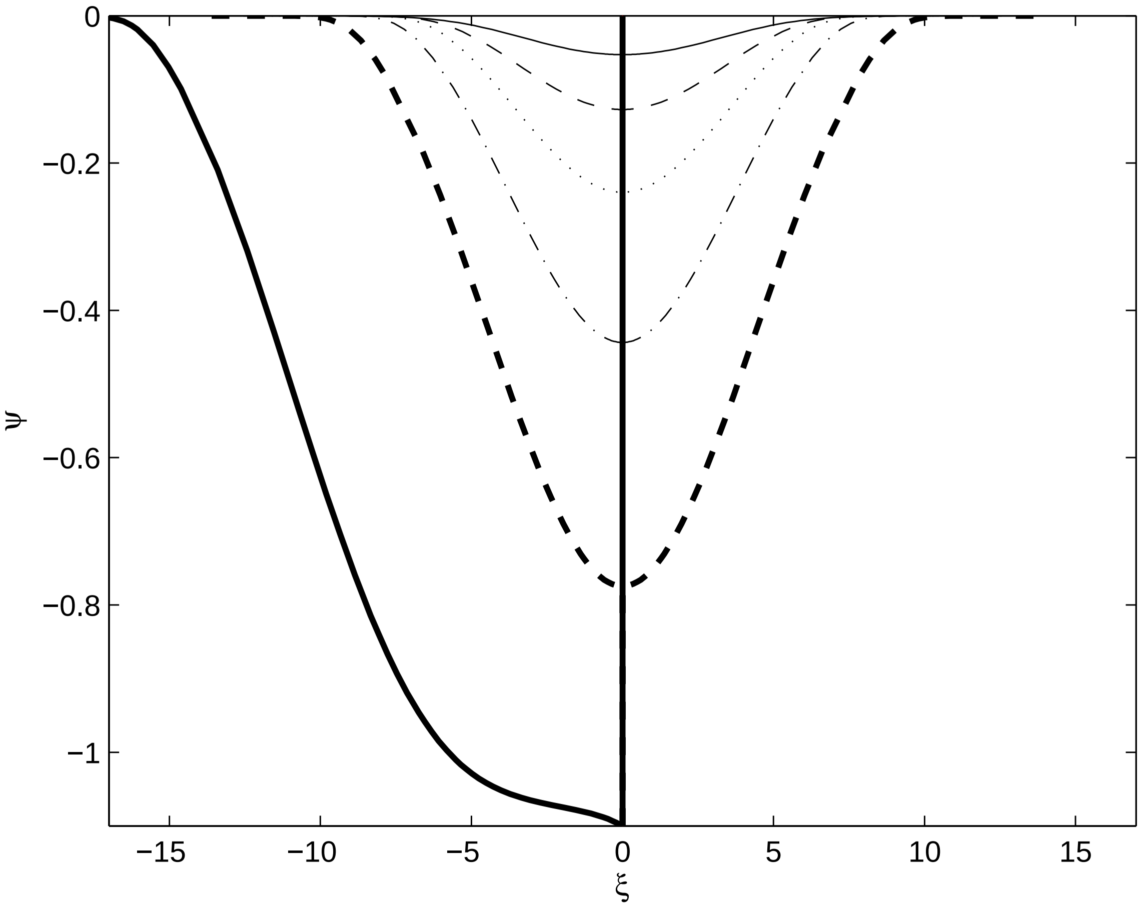

Figure 4.28: Electrostatic potential ψ vs ξ. The parameters of Figure 4.27 and θ=15^o (—), 20^o (- - -), 25^o (...), 30^o (- . -), 33^o (— — —), 33.46808^o(—) for double layer.

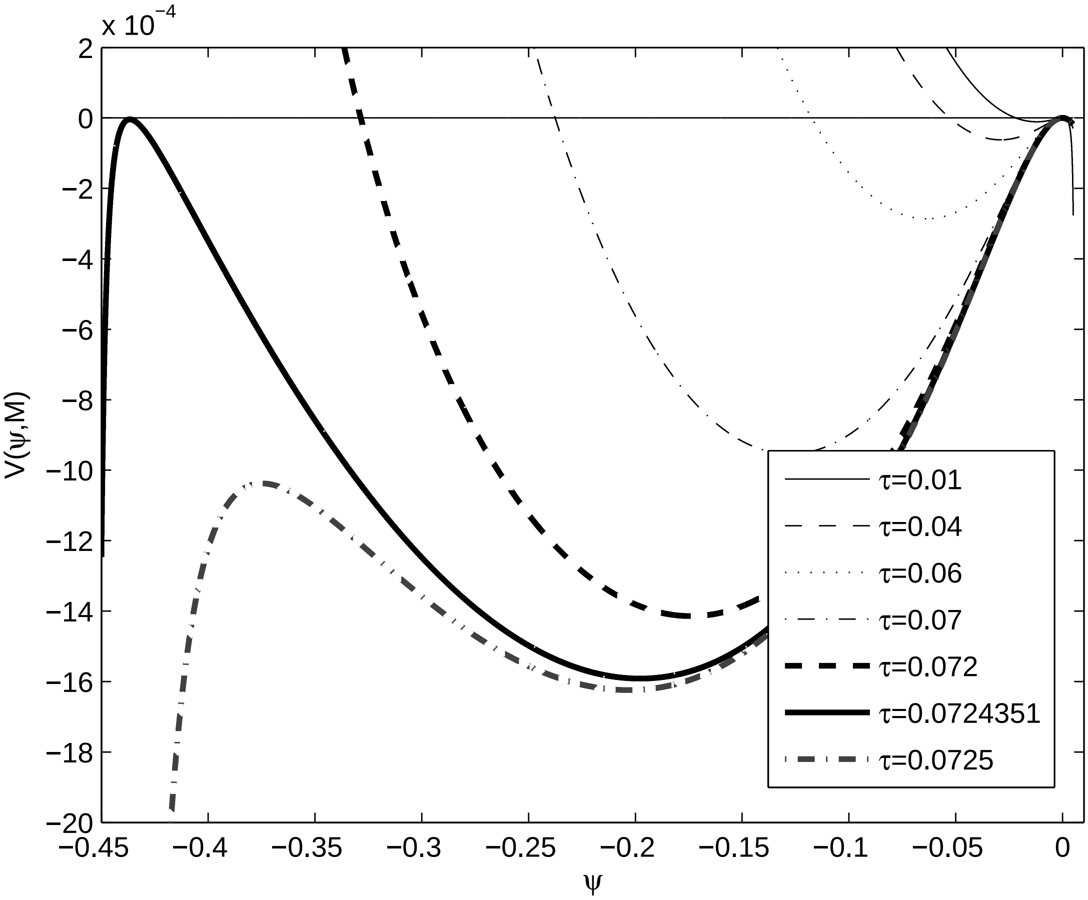

Figure 4.29: Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ. The parameters are f=0.1, g=0.1, θ=15^o, M=0.93 and τ=0.01, 0.04, 0.06, 0.07, 0.072, 0.0724351, 0.0725.

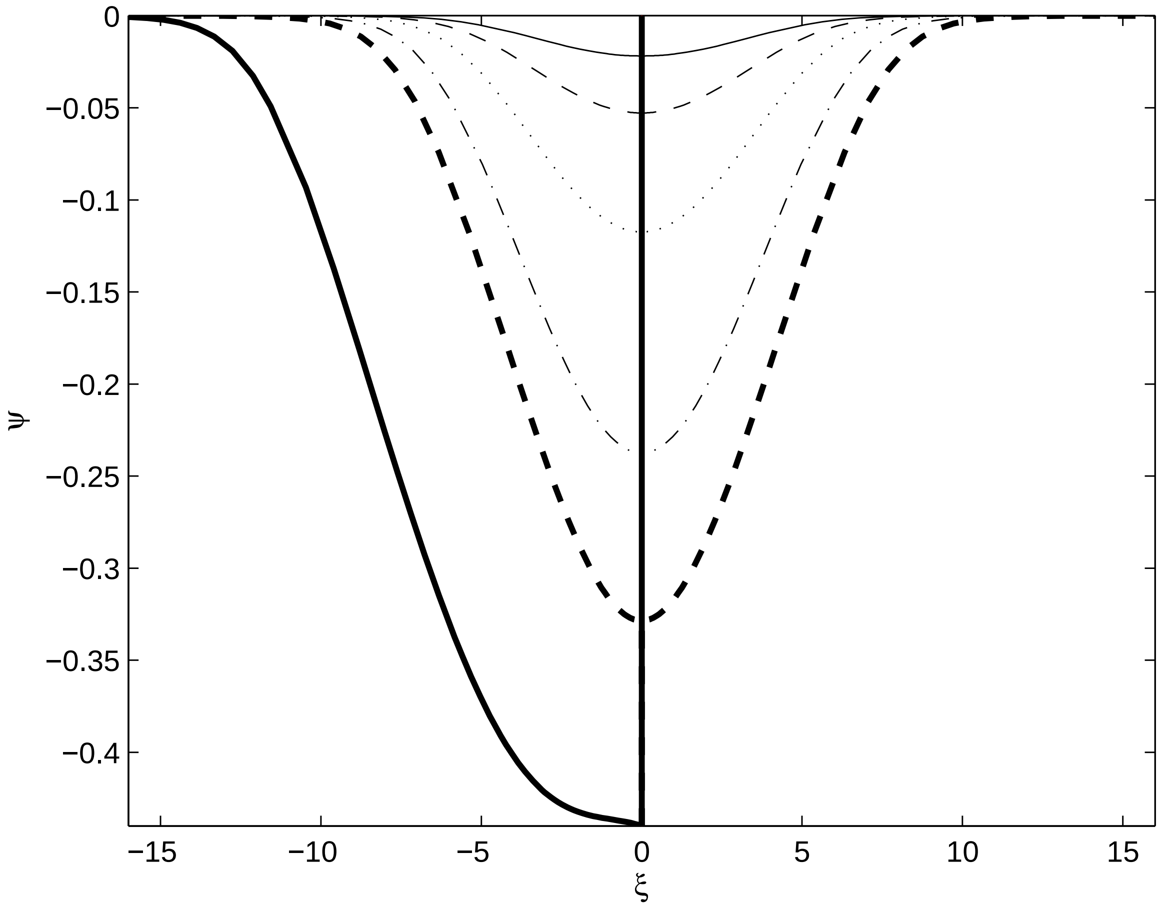

Figure 4.30: Electrostatic potential ψ vs ξ. The parameters of Figure 4.29 and τ=0.01 (—), 0.04 (- - -), 0.06 (...), 0.07 (- . -), 0.072 (— — —), 0.0724351 (—) for double layer.

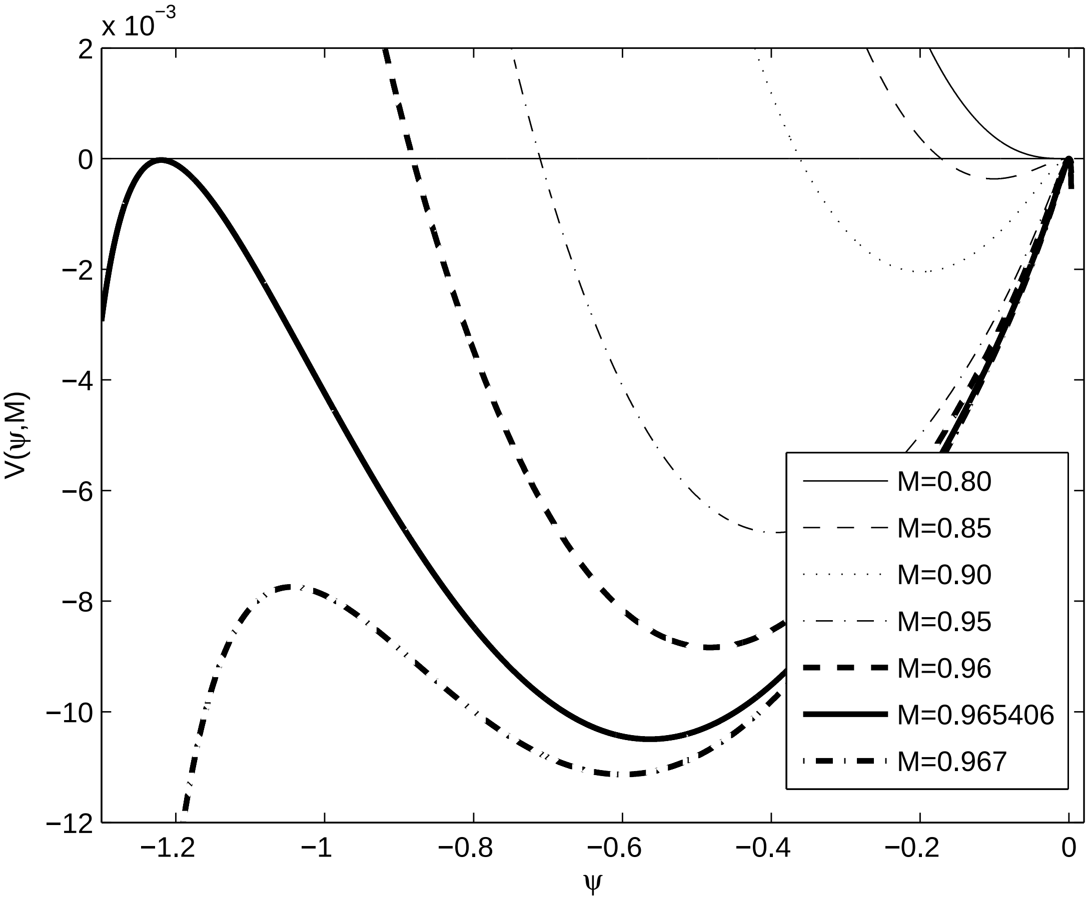

Figure 4.31: Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ. The parameters are f=0.1, g=0.05, θ=35^o, τ=0.04 and M=0.80, 0.85, 0.90, 0.95, 0.96, 0.965406, 0.967.

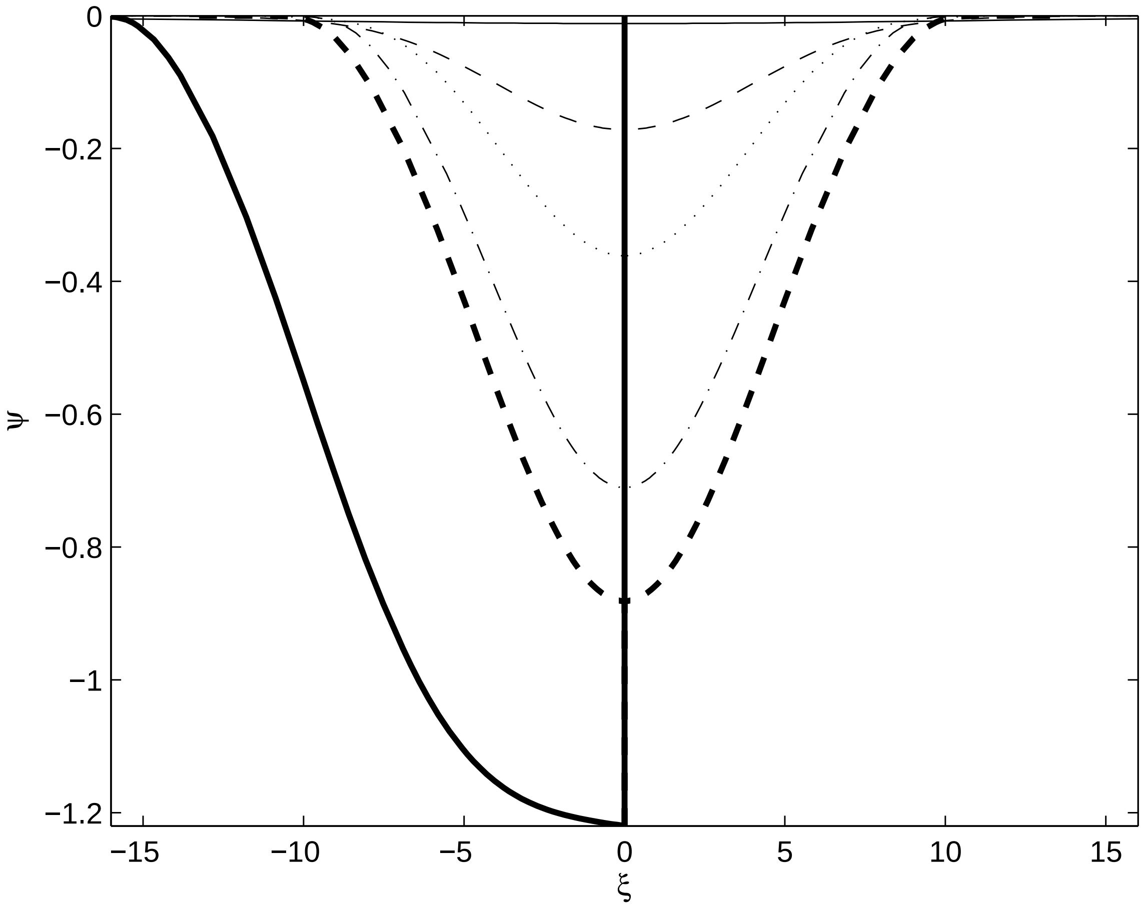

Figure 4.32: Electrostatic potential ψ vs ξ. The parameters of Figure 4.31 and M=0.80 (—), 0.85 (- - -), 0.90 (...), 0.95 (- . -) 0.96 (— — —) and 0.965406 (—).

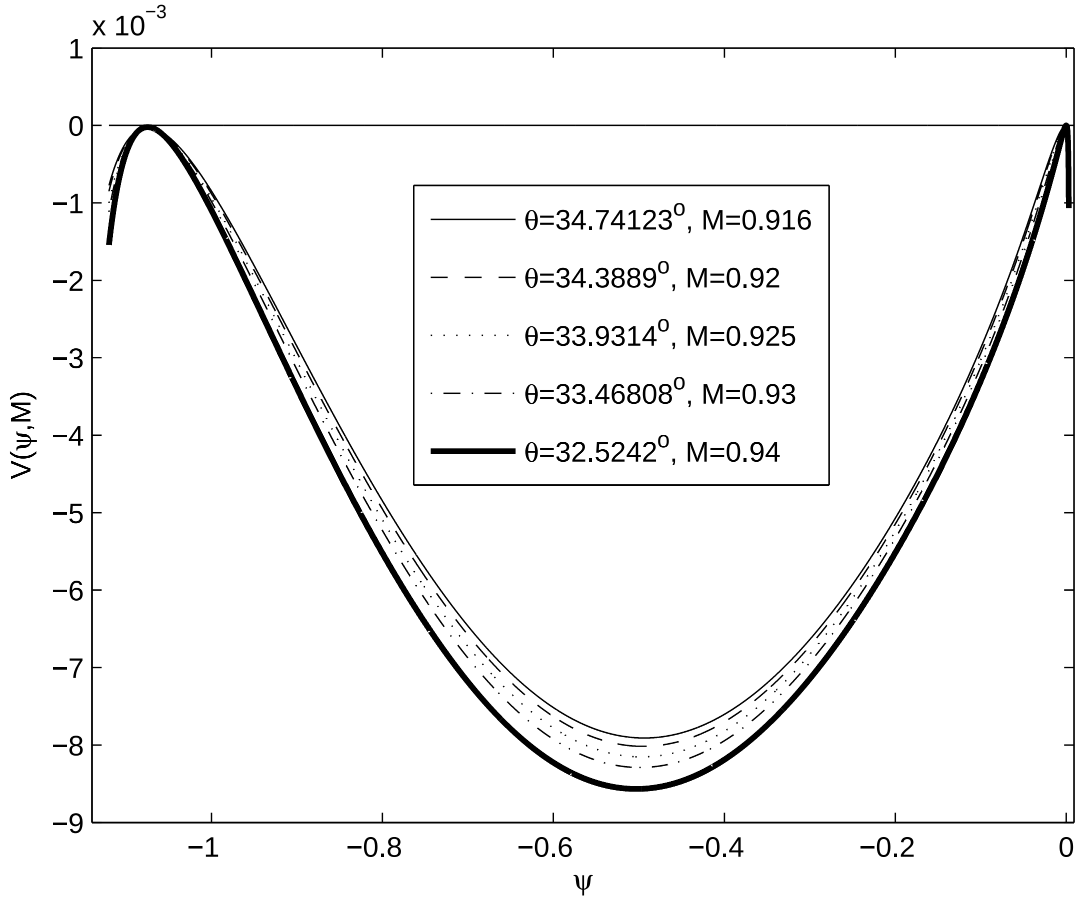

Figure 4.33: Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ. The parameters are f=0.1, g=0.1 for θ=34.74123^o and M=0.916, θ=34.3889^o and M=0.92, θ=33.9314^o and M=0.925, θ=33.46808^o and M=0.93, θ=32.5242^o and M=0.94.

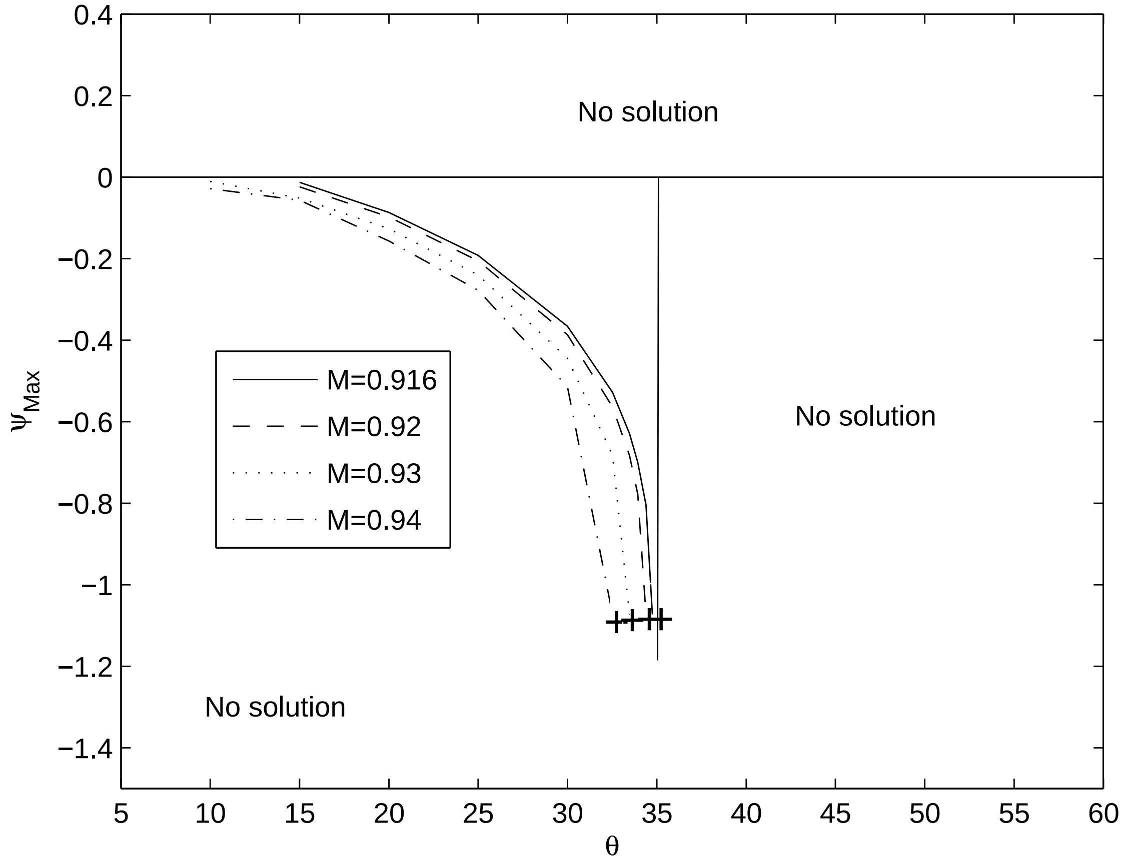

Figure 4.34: The maximum electrostatic potential ψ_{Max} vs θ. The parameters of Figure 4.33 and M=0.916, 0.92, 0.925, 0.93, 0.94.

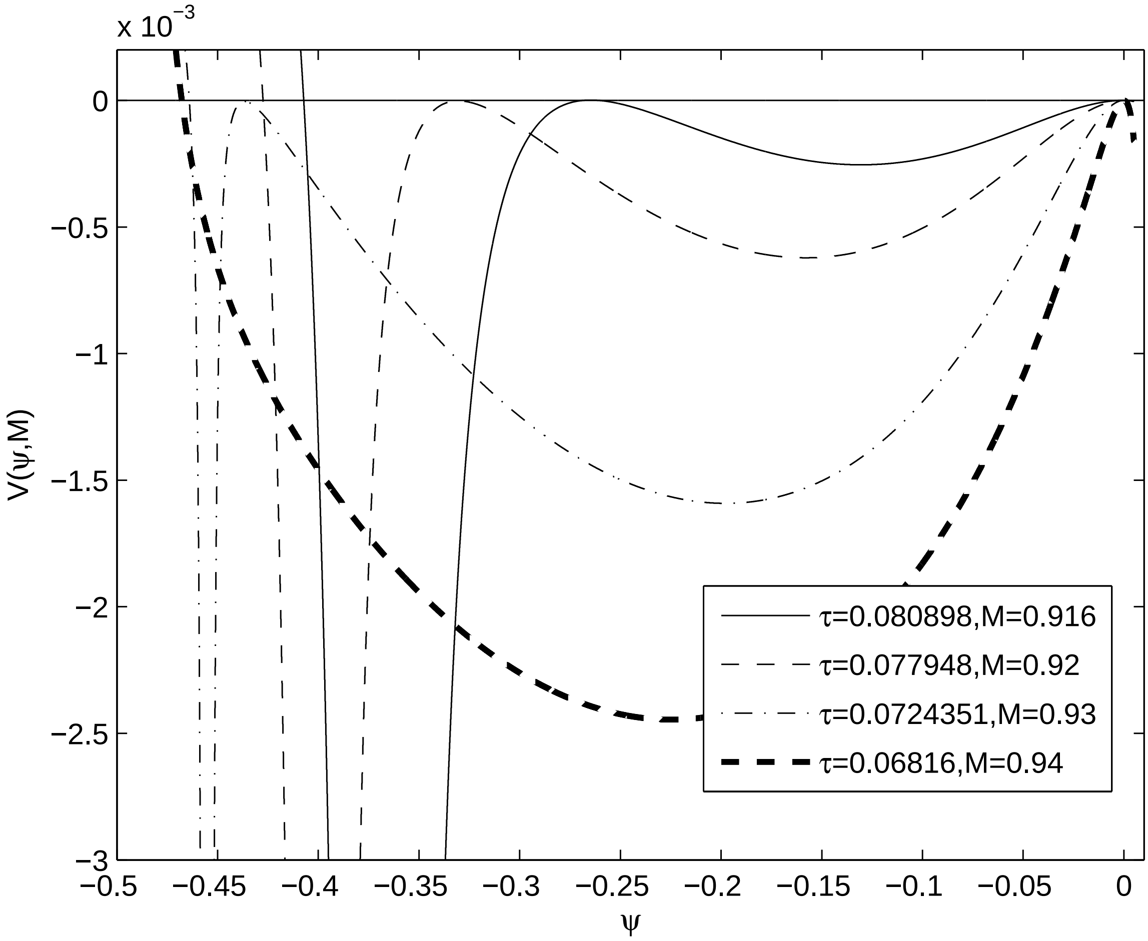

Figure 4.35: Sagdeev potential, $V(\psi, M)$ vs the normalized potential ψ. The parameters are g=0.1, f=0.1, θ=15^o and M=0.916 and τ=0.080898, M=0.92 and τ=0.077948, M=0.93 and τ=0.0724351, M=0.94 and τ=0.06816- soliton solution only.

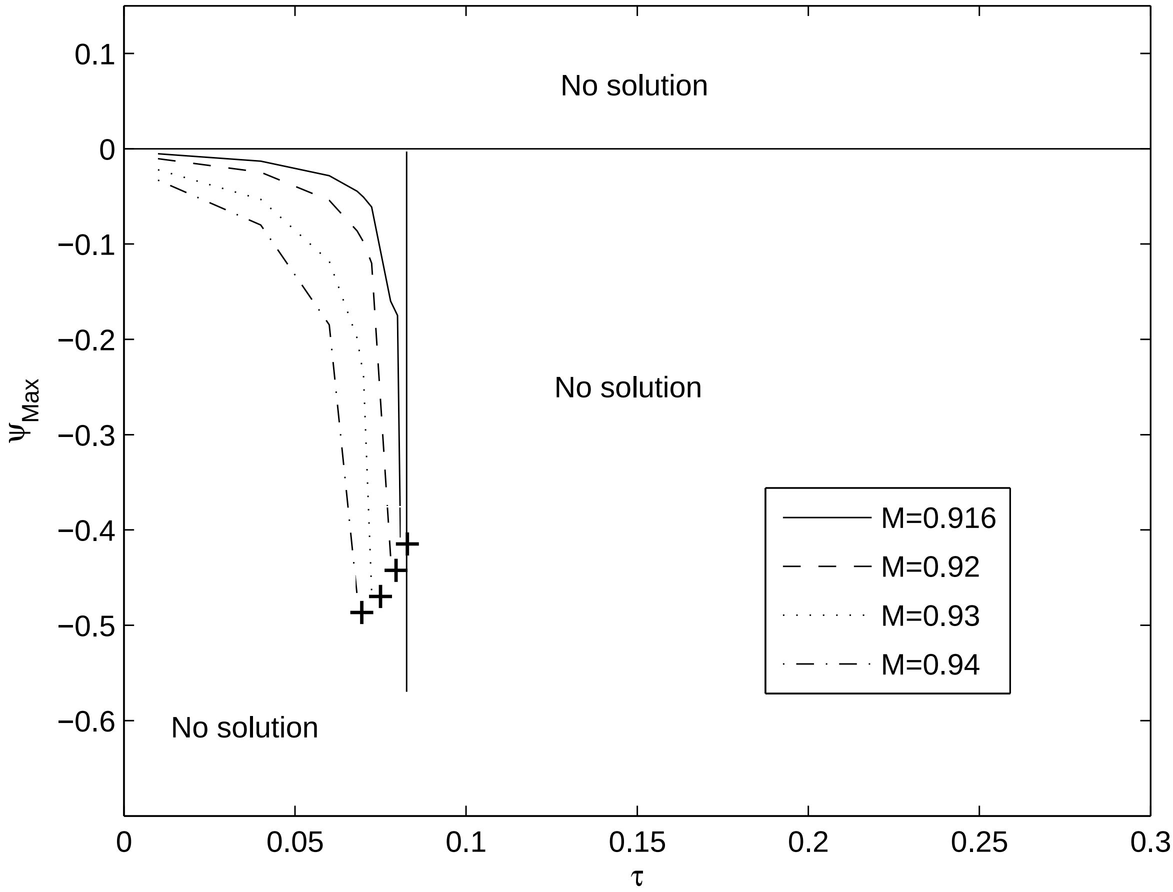

Figure 4.36: The maximum electrostatic potential ψ_{Max} vs τ. The parameters of Figure 4.35 and M=0.916, 0.92, 0.93, 0.94- soliton solution only.

Chapter 5

Summary and conclusion

In this chapter, the summary and conclusion of our investigations on nonlinear low frequency wave phenomena in space plasmas are presented. This study investigated in detail the nonlinear electrostatic structures in a magnetized plasma propagating along the auroral magnetic field lines of the Earth's magnetosphere, as reported by several spacecraft missions (i.e. S3-3, FREJA, POLAR, FAST, CLUSTER and WIND). Nonlinear low frequency (ion-acoustic) electrostatic modes have been detected in several regions of the Earth's magnetosphere (Temerin et al. 1982, Bostrom et al. 1988). These waves were categorized as low frequency due to the presence of stationary ion species (which are almost 10^3 times heavier than an electron). Several theoretical studies have been undertaken to study the nonlinear structures in multicomponent plasmas.

Chapter 3 presents a plasma model that can describe the evolution of solitons and double layers in the auroral zone of the Earth's magnetosphere, with two Boltzmann electrons distribution and cold ions fluid. The present study is an extension of the earlier work carried out by Berthomier et al. (1998), and Baluku et al. (2010), by including the magnetized effect, using the Sagdeev potential techniques. Analytical investigations of plasma consisting of Boltzmann distribution of two electron species and cold ions governed by fluid dynamic equations, were performed. Consequently, a numerical investigation of the Sagdeev potential amplitude on different plasma parameters such as Mach number, cool to hot electron density ratio, propagation angle and cool to hot electron temperature ratio, were done.

This model supports the negative potential ion-acoustic solitons and double layers, and which were found to exist only in the subsonic Mach numbers regime. In contrast as shown (Berthomier et al. 1998; Baluku et al. 2010), for the case of unmagnetized plasma, these negative potential nonlinear structures can exist only in the supersonic Mach number regime. The amplitude of the ion-acoustic solitons increases with Mach number, increased angle of propagation, cool electron density, and cool to hot electron temperature ratio.

The study was extended to investigate the finite ions' temperature effects on plasma consisting of two distinct groups of Boltzmann distributions of electrons and ions fluid. Assuming quasi-neutrality condition, the Sagdeev pseudo-potential technique was used to obtain the nonlinear localized solution and further investigation of the soliton characteristics was done to obtain critical Mach numbers for the existence of the soliton solutions. The inclusion of adiabatic ion temperature allows the nonlinear structure to exist for both subsonic and supersonic Mach number regimes. The double layers exist at a lower angle of propagation as hot ion temperature is increased. The amplitude of ion-acoustic solitary waves increases with Mach number, cool electron density, propagating angle and decreases with ions temperature. The present results concur with the Viking satellite observations in the auroral region.

This study was motivated by Cairn's nonthermal velocity distribution model for the energetic hot electron species in the study of soliton structures with density depletions observed by FREJA and Viking satellites in auroral regions of the Earth's Magnetosphere (Cairns et al. 1995). The theoretical investigation of solitary waves and double layers in auroral plasmas with two temperature electron population was also conducted. The effect of energetic hot electron species on the magnetized plasma model consisting of cold ions fluid, Boltzmann distribution of cool electrons and non-thermal distribution of hot energetic electron species was investigated. A detailed description of where the limitations in parameter space originate from, in terms of the two sonic points (lower and upper limit) and the occurrence of double layers was provided.

Chapter 4 described the finite amplitude, ion-acoustic solitary waves in magnetized three-components plasma consisting of cold oxygen ion beams, hot protons and cool electrons. The electrons and protons were considered as point particles and their density distribution taken as Boltzmann, while the heavy ions component was considered as a fluid. Using the Sagdeev potential technique and assuming charge neutrality condition, the investigation showed the evolution of only positive potential solitons.

The speed of obliquely propagating soliton was found only at subsonic Mach number region without a beam. Then the inclusion of beam velocity to the plasma model extended the solitons existence domain to both subsonic and supersonic Mach number regimes. Subsequently, numerical investigations of the Sagdeev potential and electrostatic potential variations were done on the following plasma parameters: Mach number, propagation angle and hot proton density.

Furthermore, the occurrence of nonlinear low frequency waves in a multi components plasma made up of a magnetized cold oxygen-ions fluid, Maxwellian cool ion species and Boltzmann distributions of cool and hot electron population were studied, using the Sagdeev pseudo-potential technique. This model is a modification of Model 1 (Section 3.1) by including second ion species (Boltzmann distributed), likewise it is also a modification of three-component plasma model in Section 4.1 by including additional Boltzmann electron distribution. The inclusion of additional species in this plasma model was studied and the conditions under which the soliton and double layer solutions can exist were found both analytically and numerically. The theoretical analysis showed that the Mach number regime was found to exist only in the subsonic domain. This model also supports the negative potential ion-acoustic solitons and double layers, which were found to exist only in the subsonic Mach numbers regime.

The Sagdeev potential variations were plotted on Mach numbers, electron density, propagation angle, cool to hot temperature ratio and cool ions contributions, and also showed the electrostatic potential profile. This investigation has shown that the additional species has a lot of influence on the nonlinear structures.

The theoretical results were compared to the actual satellite measurements. The findings provide good agreement.

Appendices

Appendix A

Algebraic expression for the Sagdeev potential in a magnetized plasma with cold ions and two temperature electrons

The Boltzmann distribution is assumed for the densities and temperature of the cool (N_c, T_c) and hot (N_h, T_h) electron species and are given as follows:

cool electrons

$$N_c = N_{c0} \exp(\frac{e\phi}{T_c}). \tag{A.1}$$

hot electrons

$$N_h = N_{h0} \exp(\frac{e\phi}{T_h}). \tag{A.2}$$

Cold ions (described by the fluid equations)

continuity equation

$$\frac{\partial N_i}{\partial t} + \nabla(N_i V_i) = 0. \tag{A.3}$$

momentum equation

$$\left(\frac{\partial}{\partial t} + \mathbf{V_i}.\nabla\right)\mathbf{V_i} = -\frac{e\nabla\phi}{m_i} + e\frac{\mathbf{V}_i \times \mathbf{B}_o}{m_i c}, \tag{A.4}$$

eqations (A.1)-(A.4) are unnormalized.

Normalized set of equations

$$n_c = \frac{n_{c0}}{n_{i0}} \exp\left(\frac{e\phi}{T_{eff}}.\frac{T_{eff}}{T_c}\right)$$

$$n_c = f \exp(\alpha_c \psi) \tag{A.5}$$

$$n_h = \frac{n_{h0}}{n_{i0}} \exp\left(\frac{e\phi}{T_{eff}}.\frac{T_{eff}}{T_h}\right)$$

$$n_h = (1-f) \exp(\alpha_h \psi) \tag{A.6}$$

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i v_i) \tag{A.7}$$

$$\frac{\partial v_i}{\partial t} + v_i \nabla v_i = -\nabla\psi + v_i \times z \tag{A.8}$$

use transformation

$$\xi = (\alpha x + \gamma z - Mt)/M \tag{A.9}$$

The quasi-neutrality condition is

$$n_c + n_h = n_i \tag{A.10}$$

from equations (A.7)-(A.8)

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_x)}{\partial x} + \frac{\partial (n_i v_z)}{\partial z} = 0 \tag{A.11}$$

$$\frac{\partial v_x}{\partial t} + (v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z}) v_x = -\frac{\partial \psi}{\partial x} + v_y \tag{A.12}$$

$$\frac{\partial v_y}{\partial t} + (v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z}) v_y = -v_x \tag{A.13}$$

$$\frac{\partial v_z}{\partial t} + (v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z})v_z = -\frac{\partial \psi}{\partial z} \tag{A.14}$$

from equation (A.9)

$$\frac{\partial \xi}{\partial x} = \alpha/M, \frac{\partial \xi}{\partial z} = \gamma/M, \frac{\partial \xi}{\partial t} = -1, \frac{\partial}{\partial t} = -\frac{\partial}{\partial \xi}, \frac{\partial}{\partial x} = \alpha/M\frac{\partial}{\partial \xi}, \frac{\partial}{\partial z} = \gamma/M\frac{\partial}{\partial \xi}, \tag{A.15}$$

equation (A.15) into (A.11)

$$-\frac{dn_i}{d\xi} + \frac{\alpha}{M}\frac{d(n_i v_x)}{d\xi} + \frac{\gamma}{M}\frac{d(n_i v_z)}{d\xi} = 0 \tag{A.16}$$

$$-M\frac{dn_i}{d\xi} + \alpha\frac{d(n_i v_x)}{d\xi} + \gamma\frac{d(n_i v_z)}{d\xi} = 0 \tag{A.17}$$

$$\frac{dn_i}{d\xi}(-M + \alpha v_x + \gamma v_z) = 0 \tag{A.18}$$

$$\frac{d}{d\xi}(L_v n_i) = 0 \tag{A.19}$$

where

$$L_v = -M + \alpha v_x + \gamma v_z$$

equation (A.15) into (A.12)

$$-\frac{dv_x}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_x = -\frac{\alpha}{M}\frac{d\psi}{d\xi} + v_y. \tag{A.20}$$

$$-M\frac{dv_x}{d\xi} + \alpha v_x\frac{dv_x}{d\xi} + \gamma v_z\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y \tag{A.21}$$

$$(-M + \alpha v_x + \gamma v_z)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y \tag{A.22}$$

$$L_v\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y \tag{A.23}$$

equation (A.15) into (A.13)

$$-\frac{dv_y}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_y = -v_x. \tag{A.24}$$

$$-M\frac{dv_y}{d\xi} + \alpha v_x\frac{dv_y}{d\xi} + \gamma v_z\frac{dv_y}{d\xi} = -Mv_x \tag{A.25}$$

$$(-M + \alpha v_x + \gamma v_z)\frac{dv_y}{d\xi} = -Mv_x \tag{A.26}$$

$$L_v\frac{dv_y}{d\xi} = -Mv_x \tag{A.27}$$

equation(A.15) into (A.14)

$$-\frac{dv_z}{d\xi} + (\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi})v_z = -\frac{\gamma}{M}\frac{d\psi}{d\xi}. \tag{A.28}$$

$$-M\frac{dv_z}{d\xi} + \alpha v_x\frac{dv_z}{d\xi} + \gamma v_z\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}. \tag{A.29}$$

$$(-M + \alpha v_x + \gamma v_z)\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi} \tag{A.30}$$

$$L_v\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}. \tag{A.31}$$

from equation (A.18)

$$\frac{dn_i}{d\xi}(-M + \alpha v_x + \gamma v_z) = 0 \tag{A.32}$$

$\xi = \infty$, $n_i = 1$, $v_z = 0$,$v_x = 0$

$$n_i(-M + \alpha v_x + \gamma v_z) = C \tag{A.33}$$

$$C = -M$$

$$n_i(-M + \alpha v_x + \gamma v_z) = -M$$

$$-M + \alpha v_x + \gamma v_z = -\frac{M}{n_i} \tag{A.34}$$

$$L_v = -\frac{M}{n_i} \tag{A.35}$$

from equation(A.30) we have

$$-\frac{M}{n_i}\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi} \tag{A.36}$$

$$\frac{dv_i}{d\xi} = \frac{\gamma n_i}{M}\frac{d\psi}{d\xi} \tag{A.37}$$

from (A.35) we have

$$-M + \alpha v_x + \gamma v_z = -\frac{M}{n_i}$$

differentiate w.r.t $d\xi$ we have

$$0 + \alpha\frac{dv_x}{d\xi} + \gamma\frac{dv_z}{d\xi} = \frac{M}{n_i^2}\frac{dn_i}{d\xi} \tag{A.38}$$

$$\frac{dv_x}{d\xi} = -\frac{\gamma^2 n_i}{\alpha M}\frac{d\psi}{d\xi} + \frac{M}{\alpha n_i^2}\frac{dn_i}{d\xi} \tag{A.39}$$

from equation (A.10)

$$n_i = n_c + n_h$$

$$f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi)$$

integrate equation (A.40)

$$\int\frac{dv_x}{d\xi}d\xi + \frac{\gamma^2}{M}\int(f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi))\frac{d\psi}{d\xi}d\xi = \int\frac{M}{n_i^2}\frac{dn_i}{d\xi}d\xi. \tag{A.40}$$

$$\alpha v_x + \frac{\gamma^2}{M}(\frac{f}{\alpha_c}\exp(\alpha_c\psi) + \frac{(1-f)}{\alpha_h}\exp(\alpha_h\psi)) = -\frac{m}{n_i} + C \tag{A.41}$$

$v_x = 0$, $n_i = 1$, $\psi = 0$

$$\frac{\gamma^2 f}{\alpha_c M} + \frac{\gamma^2(1-f)}{\alpha_h M} + M = C$$

$$\alpha v_x = M - \frac{M}{n_i} - \frac{\gamma^2}{M}(\frac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1))$$

$$v_x = \frac{1}{\alpha}[M - \frac{M}{n_i} - \frac{\gamma^2}{M}[\frac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1)]] \qquad \text{(A.42)}$$

from equation (A.22)

$$\begin{aligned}
(-M + \alpha v_x + \gamma v_z)\frac{dv_x}{d\xi} &= -\alpha\frac{d\psi}{d\xi} + Mv_y \\
-\frac{M}{n_i}\frac{dv_x}{d\xi} &= -\alpha\frac{d\psi}{d\xi} + Mv_y \\
-\frac{M}{n_i}\left[-\frac{\gamma^2 n_i}{\alpha M}\frac{d\psi}{d\xi} + \frac{M}{\alpha n_i^2}\frac{dn_i}{d\xi}\right] &= -\alpha\frac{d\psi}{d\xi} + Mv_y \\
\frac{\gamma^2}{\alpha}\frac{d\psi}{d\xi} - \frac{M^2}{\alpha n_i^3}\frac{dn_i}{d\xi} + \alpha\frac{d\psi}{d\xi} &= Mv_y \\
(\frac{\alpha^2+\gamma^2}{\alpha M})\frac{d\psi}{d\xi} - \frac{M^2}{\alpha n_i^3}\frac{dn_i}{d\xi} &= v_y \qquad \text{(A.43)}
\end{aligned}$$

$$\alpha^2 + \gamma^2 = \sin^2\theta + \cos^2\theta = 1$$

from equation (A.26)

$$\begin{aligned}
(-M + \alpha v_x + \gamma v_z)\frac{dv_y}{d\xi} &= -Mv_x \\
-\frac{M}{n_i}\frac{dv_y}{d\xi} &= -Mv_x \\
\frac{dv_y}{d\xi} &= n_i v_x. \qquad \text{(A.44)}
\end{aligned}$$

equation (A.43) into (A.44)

$$\frac{d}{d\xi}\left[\frac{1}{\alpha M}\frac{d\psi}{d\xi} - \frac{m}{\alpha n_i^3}\frac{dn_i}{d\xi}\right] \quad = \quad n_i v_x.$$

$$\frac{d}{d\xi}\left[(\frac{1}{\alpha M})\frac{d\psi}{d\xi} - \frac{m^2}{n_i^3}\frac{dn_i}{d\xi}\right] = n_i\left[\frac{1}{\alpha}\left(M - \frac{M}{n_i} - \frac{\gamma^2}{M}[\frac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1)\right.\right.$$

$$\frac{d}{d\xi}\left[\frac{d}{d\xi}(\psi + \frac{M^2}{2n_i^2})\right] = M^2(n_i - 1) + n_i\gamma^2\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1)\right] \quad (A.45)$$

let

$$t = \psi + \frac{M^2}{2n_i^2}.$$

$$\frac{dt}{d\xi} = \frac{d\psi}{d\xi} - \frac{M^2}{n_i^3}\frac{dn_i}{d\xi}. \quad (A.46)$$

$$n_i = n_c + n_h.$$

$$n_i = f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi).$$

$$\frac{dn_i}{d\xi} = \left[\alpha_c f\exp(\alpha_c\psi) + \alpha_h(1-f)\exp(\alpha_h\psi)\right]\frac{d\psi}{d\xi}.$$

$$\frac{dt}{d\xi} = \frac{d\psi}{d\xi} - \frac{M^2}{n_i^3}\alpha_c f\exp(\alpha_c\psi) + \alpha_h(1-f)\exp(\alpha_h\psi)\frac{d\psi}{d\xi}.$$

$$\frac{dt}{d\xi} \quad = \quad \left(1 - M^2\frac{(\alpha_c f\exp(\alpha_c\psi) + \alpha_h(1-f)\exp(\alpha_h\psi))}{(f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi))^3}\right)\frac{d\psi}{d\xi}. \quad (A.47)$$

multiply both side of equation (A.46) by $2\frac{dt}{d\xi}$ and integrate

$$\int 2\frac{dt}{d\xi}\frac{d}{d\xi}(\frac{dt}{d\xi})d\xi =$$
$$\int 2\left[M^2(n_i - 1) + n_i\gamma^2\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1)\right)\right]\frac{dt}{d\xi}d\xi.$$

$$\left(\frac{dt}{d\xi}\right)^2 =$$
$$2\left[\int M^2(n_i-1)\frac{dt}{d\xi}d\xi + \gamma^2\int n_i\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)\right]\frac{dt}{d\xi}.d\xi.$$

$$\begin{aligned}
&\frac{1}{2}\left[1-M^2\left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^3}\right)\right]^2\left(\frac{d\psi}{d\xi}\right)^2\\
&= M^2\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h\psi}(\exp(\alpha_h\psi)-1)\right]-M^2\psi-M^4\left[-\frac{1}{n_i}+\frac{1}{2n_i^2}+\frac{1}{2}\right]\\
&\quad -\gamma^2\int f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi)\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right]d\psi\\
&\quad +M^2\gamma^2\int\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right]\\
&\quad \left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^2}\right)d\psi.
\end{aligned}$$

let

$$\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)=p$$
$$f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi)=\frac{dp}{d\psi}. \tag{A.48}$$

$$\begin{aligned}
&\frac{1}{2}\left[1-M^2\left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^3}\right)\right]^2\left(\frac{d\psi}{d\xi}\right)^2\\
&= -\frac{M^4}{2n_i^2}(1-n_i)^2-M^2\psi+M^2\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h\psi}(\exp(\alpha_h\psi)-1)\right]\\
&\quad -\gamma^2\int\frac{dp}{d\psi}.p.d\psi \quad +M^2\gamma^2\int\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right]\\
&\quad \left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^2}\right)d\psi.
\end{aligned}$$

now, let

$$g=f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi).$$
$$\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)d\psi=dg. \tag{A.49}$$

$$\frac{1}{2}\left[1-M^2\left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^3}\right)\right]^2\left(\frac{d\psi}{d\xi}\right)^2$$
$$=-\frac{M^4}{2n_i^2}(1-n_i)^2-M^2\psi+M^2\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h\psi}(\exp(\alpha_h\psi)-1)\right]-\gamma^2\left[\frac{p^2}{2}\right]$$
$$+M^2\gamma^2\int\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right]\frac{dg}{g^2}.$$

Also,let

$$dv=\frac{dg}{g^2}. \tag{A.50}$$

$$v=-\frac{1}{g}. \tag{A.51}$$

$$\int pdv=pv-\int vdp. \tag{A.52}$$

we have

$$\frac{1}{2}\left[1-M^2\left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^3}\right)\right]^2\left(\frac{d\psi}{d\xi}\right)^2$$
$$=-\frac{M^4}{2n_i^2}(1-n_i)^2-M^2\psi+M^2\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h\psi}(\exp(\alpha_h\psi)-1)\right]$$
$$-\frac{\gamma^2}{2}\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right]^2$$
$$+M^2\gamma^2\left[\frac{\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)}{f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi)}\right]+M^2\gamma^2\psi.$$

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2+V(\psi)=0 \tag{A.53}$$

$$V(\psi)=-\frac{1}{\left(1-M^2\left(\frac{\alpha_c f\exp(\alpha_c\psi)+\alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi))^3}\right)\right)^2}\times$$
$$\left\{\begin{array}{l}-\frac{M^4}{2n_i^2}(1-n_i)^2-M^2(1-\gamma^2)\psi\\ +M^2\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)\\ -\frac{\gamma^2}{2}\left[\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)\right]^2\\ +M^2\gamma^2\left[\frac{\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi)-1)}{f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi)}\right].\end{array}\right\}$$

Appendix B

Algebraic expression for the Sagdeev potential in a magnetized plasma with adiabatic ion and two temperature electrons

The Boltzmann distribution is assumed for the densities and temperature of the cool(n_c, T_c) and hot (n_h, T_h) electron species and are given in normalized form as follows:

cool electrons

$$n_c = \frac{n_{c0}}{n_{i0}} \exp\left(\frac{e\phi}{T_{eff}}.\frac{T_{eff}}{T_c}\right)$$

$$n_c = f \exp(\alpha_c \psi) \tag{B.1}$$

hot electrons

$$n_h = \frac{n_{h0}}{n_{i0}} \exp\left(\frac{e\phi}{T_{eff}}.\frac{T_{eff}}{T_h}\right)$$

$$n_h = (1-f) \exp(\alpha_h \psi) \tag{B.2}$$

The hot adiabatic ions is described by the fluid equations.

Continuity equation:

$$\frac{\partial n_i}{\partial t} + \nabla(n_i V_i) = 0.$$

Momentum equation:

$$\left(\frac{\partial}{\partial t} + \mathbf{V_i}.\nabla\right)\mathbf{V_i} = -\frac{e\nabla\phi}{m_i} + e\frac{\mathbf{V}_i \times \mathbf{B}_o}{m_i c} - \frac{1}{n_i m_i}\nabla\tilde{p}_i.$$

equation of state

$$\frac{\partial p_i}{\partial t} + V_i\nabla.p_i + \delta p_i.\nabla V_i = 0$$

from the equation of state

$$V_i.\nabla p_i = 0$$

then

$$\frac{1}{p}\frac{dp_i}{dt} = -\frac{\delta}{n}\frac{dn_i}{dt},$$

where

$$\delta\nabla.V_i = \frac{1}{n}\frac{dn_i}{dt}$$

then, integrate

$$p_i = p_{i0}\left(\frac{n_o}{n_{i0}}\right)^{\delta}$$

if N is the number of degrees of freedom, δ is given by

$$\delta = \frac{N+2}{N}$$

from continuty and momentum equations (normalized)

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_x)}{\partial x} + \frac{\partial(n_i v_z)}{\partial z} = 0 \tag{B.3}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial\psi}{\partial x} + v_y - \frac{\sigma}{n_i}\frac{\partial}{\partial x}.(n_i)^{5/3} \tag{B.4}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_y = -\frac{\partial\psi}{\partial y} - v_x \tag{B.5}$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial\psi}{\partial z} - \frac{\sigma}{n_i}\frac{\partial}{\partial z}.(n_i)^{5/3} \tag{B.6}$$

then, the set of equations are closed with the quasi-neutrality condition

$$n_i = n_c + n_h = f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi) \tag{B.7}$$

stationary frame

$$\xi = (\alpha x + \gamma z - Mt)/M \tag{B.8}$$

$$\frac{\partial\xi}{\partial x} = \frac{\alpha}{M}, \frac{\partial\xi}{\partial z} = \frac{\gamma}{M}, \frac{\partial\xi}{\partial t} = -1$$

from equation (3)

$$-\frac{dn_i}{d\xi} + \frac{\alpha}{M}\frac{dn_i v_x}{d\xi} + \frac{\gamma}{M}\frac{dn_i v_x}{d\xi} = 0$$

$$M\frac{dn_i}{d\xi} = \frac{dn_i}{d\xi}(\alpha v_x + \gamma v_z)$$

integrate with the boundary condition

$$\xi \to 0, n_i \to 1, \psi = 0, v_x = v_z = 0$$

$$M = -C$$

$$M n_i - M = n_i(\alpha v_x + \gamma v_z)$$

$$\alpha v_x + \gamma v_z = M\left(1 - \frac{1}{n_i}\right) \tag{B.9}$$

from equation (B.4)

$$-\frac{dv_x}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_x = -\frac{\alpha}{M}\frac{d\psi}{d\xi} + v_y - \frac{5\sigma\alpha}{3n_i^{1/3}M}\frac{dn_i}{d\xi}$$

$$-M\frac{dv_x}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y - \frac{5\sigma\alpha}{3n_i^{1/3}}\frac{dn_i}{d\xi}$$

$$\left(-M + M - \frac{M}{n_i}\right)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y - \frac{5\sigma\alpha}{3n_i^{1/3}}\frac{dn_i}{d\xi}$$

$$-\frac{M}{n_i}\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} - \frac{5\sigma\alpha}{3n_i^{1/3}}\frac{dn_i}{d\xi} + Mv_y \tag{B.10}$$

from equation (B.5)

$$-\frac{dv_y}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_y = -v_x$$

$$-M\frac{dv_y}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_y}{d\xi} = -Mv_x$$

$$\left(-M + M - \frac{M}{n_i}\right)\frac{dv_y}{d\xi} = -Mv_x$$

$$\frac{1}{n_i}\frac{dv_y}{d\xi} = v_x \tag{B.11}$$

from equation (B.6)

$$-\frac{dv_z}{d\xi}+\left(\frac{\alpha}{M}v_x\frac{d}{d\xi}+\frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_z=-\frac{\gamma}{M}\frac{d\psi}{d\xi}-\frac{5\sigma\alpha}{3n_i^{1/3}M}\frac{dn_i}{d\xi}$$

$$-M\frac{dv_z}{d\xi}+(\alpha v_x+\gamma v_z)\frac{dv_z}{d\xi}=-\gamma\frac{d\psi}{d\xi}+\frac{5\sigma\alpha}{3n_i^{1/3}}\frac{dn_i}{d\xi}$$

$$\left(-M+M-\frac{M}{n_i}\right)\frac{dv_z}{d\xi}=-\gamma\left(\frac{d\psi}{d\xi}+\frac{5\sigma\alpha}{3n_i^{1/3}}\frac{dn_i}{d\xi}\right)$$

$$\frac{M}{n_i}\frac{dv_z}{d\xi}=\gamma\left(\frac{d\psi}{d\xi}+\frac{5\sigma\alpha}{3n_i^{1/3}}\frac{dn_i}{d\xi}\right)\tag{B.12}$$

differentiate equation (B.9) w.r.t . ∂_ξ, we have

$$\alpha\frac{dv_x}{d\xi}+\gamma\frac{dv_z}{d\xi}=\frac{M}{n_i^2}\frac{dn_i}{d\xi}\tag{B.13}$$

then, substitute equation (B.12) into (B.13)

$$\alpha\frac{dv_x}{d\xi}+\gamma\left(\frac{\gamma n_i}{M}\left(\frac{d\psi}{d\xi}+\frac{5\sigma}{3n_i^{1/3}}\frac{dn_i}{d\xi}\right)\right)=\frac{M}{n_i^2}\frac{dn_i}{d\xi}$$

$$\frac{dv_x}{d\xi}=\frac{M}{\alpha n_i^2}\frac{dn_i}{d\xi}-\frac{5\gamma^2\sigma n_i^{2/3}}{3\alpha M}\frac{dn_i}{d\xi}-\frac{\gamma^2 n_i}{\alpha M}\frac{d\psi}{d\xi}\tag{B.14}$$

integrate equation (B.14)

$$\alpha\int\frac{dv_x}{d\xi}d\xi+\frac{\gamma^2}{M}\int n_i\frac{d\psi}{d\xi}d\xi=M\int\frac{1}{n_i^2}\frac{dn_i}{d\xi}d\xi-\frac{5\gamma^3\sigma}{3M}\int n_i^{2/3}\frac{dn_i}{d\xi}d\xi$$

$$\alpha v_x+\frac{\gamma^2}{M}\left(\frac{f}{\alpha_c}\exp(\alpha_c\psi)+\frac{(1-f)}{\alpha_h}\exp(\alpha_h\psi)\right)=-\frac{M}{n_i}-\frac{\gamma^2\sigma}{M}n_i^{5/3}+C$$

at

$$v_x=0, n_i=1, \psi=0$$

$$\frac{\gamma^2}{M}\left(\frac{f}{\alpha_c}+\frac{1-f}{\alpha_h}\right)+M+\frac{\gamma^2\sigma}{M}=C$$

therefore,

$$\alpha v_x = M\left(1-\frac{1}{n_i}\right)+\frac{\gamma^2\sigma}{M}(1-n_i^{5/3})-\frac{\gamma^2}{M}\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{1-f}{\alpha_h}(\exp(\alpha_c\psi)-1)\right) \tag{B.15}$$

equation (B.14) into (B.10)

$$-\frac{\gamma^2}{\alpha}\frac{d\psi}{d\xi}-\frac{M^2}{\alpha n_i^3}\frac{dn_i}{d\xi}+\frac{5\gamma^2\sigma n_i^{-1/3}}{3\alpha}\frac{dn_i}{d\xi}=-\alpha\frac{d\psi}{d\xi}-\frac{5\alpha\sigma}{3n_i^{1/3}}\frac{dn_i}{d\xi}+Mv_y$$

$$\left(\frac{\gamma^2+\alpha^2}{\alpha}\right)\frac{d\psi}{d\xi}-\frac{M^2}{\alpha n_i^3}\frac{dn_i}{d\xi}+\frac{5\sigma n_i^{-1/3}}{3}\left(\frac{\gamma^2+\alpha^2}{\alpha}\right)\frac{dn_i}{d\xi}=Mv_y$$

then

$$\gamma^2+\alpha^2=1,$$

since

$$\cos^2\theta+\sin^2\theta=1$$

we have,

$$\frac{1}{\alpha M}\left(\frac{d\psi}{d\xi}-\frac{M^2}{n_i^3}\frac{dn_i}{d\xi}+\frac{5\sigma n_i^{-1/3}}{3}\frac{dn_i}{d\xi}\right)=v_y \tag{B.16}$$

now, substitute equation (B.11) into (B.16)

$$\frac{d}{d\xi}\left(\frac{d\psi}{d\xi}-\frac{M^2}{n_i^3}\frac{dn_i}{d\xi}+\frac{5\sigma n_i^{-1/3}}{3}\frac{dn_i}{d\xi}\right)=M\alpha n_i v_x \tag{B.17}$$

$$\begin{aligned}&\frac{d}{d\xi}\left(\frac{d}{d\xi}\left(\psi+\frac{M^2}{2n_i^2}+\frac{5}{2}\sigma n_i^{2/3}\right)\right)\\&=M^2(n_i-1)+\gamma^2\sigma n_i(1-n_i^{5/3})-\gamma^2 n_i\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{1-f}{\alpha_h}(\exp(\alpha_c\psi)-1)\right)\end{aligned} \tag{B.18}$$

let

$$t=\psi+\frac{M^2}{2n_i^2}+\frac{5}{2}\sigma n_i^{2/3},$$

then

$$\frac{dt}{d\xi}=\frac{d\psi}{d\xi}-\frac{M^2}{n_i^3}\frac{dn_i}{d\xi}+\frac{5}{3}\sigma n_i^{-1/3}\frac{dn_i}{d\xi}$$

$$n_i=f\exp(\alpha_c\psi)+(1-f)\exp(\alpha_h\psi)$$

$$\frac{dn_i}{d\xi} = (\alpha_c f \exp(\alpha_c \psi) + \alpha_h (1-f) \exp(\alpha_h \psi) \frac{d\psi}{d\xi}$$

$$\frac{dt}{d\xi} = \left(1 - \frac{M^2}{n_i^3} (\alpha_c f \exp(\alpha_c \psi) + \alpha_h (1-f) \exp(\alpha_h \psi) \right.$$
$$\left. + \frac{5\sigma n_i^{-1/3}}{3} (\alpha_c f \exp(\alpha_c \psi) + \alpha_h (1-f) \exp(\alpha_h \psi) \right) \frac{d\psi}{d\xi}$$

Mutiply both sides of equation (B.18) by $2\frac{dt}{d\xi}$ and integrate

$$2 \int \frac{dt}{d\xi} . \frac{d}{d\xi} \left(\frac{dt}{d\xi} \right) d\xi = 2 \left[\int M^2 (n_i - 1) + \gamma^2 \sigma n_i (1 - n_i^{5/3}) \right.$$
$$\left. - \gamma^2 n_i \left(\frac{f}{\alpha_c} (\exp(\alpha_c \psi) - 1) + \frac{1-f}{\alpha_h} (\exp(\alpha_h \psi) - 1) \right) \right] \frac{dt}{d\xi}$$

$$\frac{1}{2} \left(\frac{dt}{d\xi} \right)^2 = - \frac{M^4}{2n_i^2} (1 - n_i)^2 - M^2 (1 - \gamma^2) \psi + M^2 \left(\frac{f}{\alpha_c} (\exp(\alpha_c \psi) - 1) + \frac{1-f}{\alpha_h} (\exp(\alpha_h \psi) - 1) \right)$$
$$+ \sigma M^2 \left(n_i^{5/3} - \frac{5n_i^{2/3}}{2} + \frac{3}{2} \right) + \gamma^2 \sigma \left(\frac{f}{\alpha_c} (\exp(\alpha_c \psi) - 1) + \frac{1-f}{\alpha_h} (\exp(\alpha_h \psi) - 1) \right)$$
$$- \gamma^2 M^2 \sigma \left(- \frac{1}{n_i} - \frac{3n_i^{2/3}}{2} + \frac{5}{2} \right) + \gamma^2 \sigma^2 \left(n_i^{5/3} - \frac{n_i^{10/3}}{2} - \frac{1}{2} \right)$$
$$- \frac{\gamma^2}{2} \left(\frac{f}{\alpha_c} (\exp(\alpha_c \psi) - 1) + \frac{1-f}{\alpha_h} (\exp(\alpha_h \psi) - 1) \right)^2$$
$$- \frac{\gamma^2 M^2}{n_i} \left(\frac{f}{\alpha_c} (\exp(\alpha_c \psi) - 1) + \frac{1-f}{\alpha_h} (\exp(\alpha_h \psi) - 1) \right)$$
$$- \gamma^2 \sigma n_i^{5/3} \left(\frac{f}{\alpha_c} (\exp(\alpha_c \psi) - 1) + \frac{1-f}{\alpha_h} (\exp(\alpha_h \psi) - 1) \right)$$

then, we obtain

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 = \frac{1}{\left(1-\frac{M^2}{n_i^3}\left(\alpha_c f(e^{\alpha_c\psi}+\alpha_h(1-f)e^{\alpha_h\psi}\right)+\frac{5\sigma}{3n_i^{1/3}}\left(\alpha_c f(e^{\alpha_c\psi}+\alpha_h(1-f)e^{\alpha_h\psi}\right)\right)^2}\times$$

$$-\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\gamma^2)\psi + M^2\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{1-f}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)$$

$$+\sigma M^2\left(n_i^{5/3}-\frac{5n_i^{2/3}}{2}+\frac{3}{2}\right)+\gamma^2\sigma\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{1-f}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)$$

$$-\gamma^2M^2\sigma\left(-\frac{1}{n_i}-\frac{3n_i^{2/3}}{2}+\frac{5}{2}\right)+\gamma^2\sigma^2\left(n_i^{5/3}-\frac{n_i^{10/3}}{2}-\frac{1}{2}\right)$$

$$-\frac{\gamma^2}{2}\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{1-f}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)^2$$

$$-\frac{\gamma^2M^2}{n_i}\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{1-f}{\alpha_h}(\exp(\alpha_h\psi)-1)\right)$$

$$-\gamma^2\sigma n_i^{5/3}\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1)+\frac{1-f}{\alpha_h}(\exp(\alpha_h\psi)-1)\right) \tag{B.19}$$

Appendix C

Algebraic expression for the Sagdeev potential in a magnetized plasma with cold ion, cool electron and nonthermal hot electron

The density and temperature of the Boltzmann distributed cool electrons (n_c, T_c) and nonthermal distributed hot electrons (n_h, T_h) in normalized form are:

cool electrons

$$n_c = \frac{n_{c0}}{n_{i0}} \exp\left(\frac{e\phi}{T_{eff}}.\frac{T_{eff}}{T_c}\right)$$

$$n_c = f \exp(\alpha_c \psi) \tag{C.1}$$

hot electrons (nonthermal)

$$n_h = \frac{n_{h0}}{n_{i0}}(1 - \beta\phi + \beta\phi^2) \exp\left(\frac{e\phi}{T_{eff}}.\frac{T_{eff}}{T_h}\right)$$

$$n_h = (1 - f)(1 - \beta\phi + \beta\phi^2) \exp(\alpha_h \psi) \tag{C.2}$$

where

$$\beta = \frac{4\alpha}{1 + 3\alpha}$$

magnetized cold ions (described by the fluid equations) (normalized)

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_x)}{\partial x} + \frac{\partial (n_i v_z)}{\partial z} = 0 \tag{C.3}$$

$$\frac{\partial v_x}{\partial t} + (v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z}) v_x = -\frac{\partial \psi}{\partial x} + v_y \tag{C.4}$$

$$\frac{\partial v_y}{\partial t} + (v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z}) v_y = -v_x \tag{C.5}$$

$$\frac{\partial v_z}{\partial t} + (v_x \frac{\partial}{\partial x} + v_z \frac{\partial}{\partial z}) v_z = -\frac{\partial \psi}{\partial z} \tag{C.6}$$

the quasi-neutrality condition

$$n_i = n_{c0} + n_{h0} = f \exp(\alpha_c \psi) + (1-f)(1 - \beta\psi + \beta\psi^2) \exp(\alpha_h \psi) \tag{C.7}$$

localized frame

$$\xi = (\alpha x + \gamma z - Mt)/M \tag{C.8}$$

$$\frac{\partial \xi}{\partial x} = \frac{\alpha}{M}, \frac{\partial \xi}{\partial z} = \frac{\gamma}{M}, \frac{\partial \xi}{\partial \xi} \partial t = -1,$$

$$\partial_t = -\partial_\xi, \partial_x = \frac{\alpha}{M} \partial_\xi, \partial_z = \frac{\gamma}{M} \partial_\xi$$

from equation (C.3)

$$-\frac{dn_i}{d\xi} + \frac{\alpha}{M} \frac{dn_i v_x}{d\xi} + \frac{\gamma}{M} \frac{dn_i v_z}{d\xi} = 0$$

$$-M \frac{dn_i}{d\xi} + \alpha \frac{dn_i v_x}{d\xi} + \gamma \frac{dn_i v_z}{d\xi} = 0$$

$$\frac{dn_i}{d\xi} (-M + \alpha v_x + \gamma v_z) = 0$$

$$\frac{d}{d\xi} (L_v n_i) = 0 \tag{C.9}$$

where

$$L_v = -M + \alpha v_x + \gamma v_z$$

from equation (C.4)

$$-\frac{dv_x}{d\xi} = \left(\frac{\alpha v_x}{M} \frac{d}{d\xi} + \frac{\gamma v_z}{M} \frac{d}{d\xi} \right) v_x = -\frac{\alpha}{M} \frac{d\psi}{d\xi} + v_y$$

$$-M \frac{dv_x}{d\xi} + \alpha v_x \frac{dv_x}{d\xi} + \gamma v_z \frac{dv_x}{d\xi} = -\alpha \frac{d\psi}{d\xi} + M v_y$$

$$(-M + \alpha v_x + \gamma v_z)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y$$

$$L_v\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y \tag{C.10}$$

from equation (C.5)

$$-\frac{dv_y}{d\xi} = \left(\frac{\alpha v_x}{M}\frac{d}{d\xi} + \frac{\gamma v_z}{M}\frac{d}{d\xi}\right)v_y = -v_x$$

$$-M\frac{dv_y}{d\xi} + \alpha v_x\frac{dv_y}{d\xi} + \gamma v_z\frac{dv_y}{d\xi} = -Mv_x$$

$$(-M + \alpha v_x + \gamma v_z)\frac{dv_y}{d\xi} = -Mv_x$$

$$L_v\frac{dv_y}{d\xi} = -Mv_x \tag{C.11}$$

from equation (C.6)

$$-\frac{dv_z}{d\xi} = \left(\frac{\alpha v_x}{M}\frac{d}{d\xi} + \frac{\gamma v_z}{M}\frac{d}{d\xi}\right)v_z = -\frac{\gamma}{M}\frac{d\psi}{d\xi}$$

$$-M\frac{dv_z}{d\xi} + \alpha v_x\frac{dv_z}{d\xi} + \gamma v_z\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}$$

$$(-M + \alpha v_x + \gamma v_z)\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}$$

$$L_v\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi} \tag{C.12}$$

now, integrate equation (C.9) with boundary conditions

$$\xi \to \infty, n_i = 1, v_x = v_z = 0$$

we have

$$n_i(-M + \alpha v_x + \gamma v_z) = C$$

$$C = M$$

then

$$-M + \alpha v_x - \gamma v_z = -\frac{M}{n_i}$$

$$L_v = -\frac{M}{n_i} \tag{C.13}$$

equation (C.13) into (C.12)

$$(-M+\alpha v_x+\gamma v_z)\frac{dv_z}{d\xi}=-\gamma\frac{d\psi}{d\xi}$$

$$\frac{M}{n_i}\frac{dv_z}{d\xi}=\gamma\frac{d\psi}{d\xi}$$

$$\frac{dv_z}{d\xi}=\frac{\gamma n_i}{M}\frac{d\psi}{d\xi} \tag{C.14}$$

differentiate equation (C.19) w.r.t ξ, we have

$$0+\alpha\frac{dv_x}{d\xi}+\gamma\frac{dv_z}{d\xi}=\frac{M}{n_i}\frac{dn_i}{d\xi}$$

$$\alpha\frac{dv_x}{d\xi}+\frac{\gamma^2 n_i}{M}=\frac{M}{n_i}\frac{dn_i}{d\xi}$$

$$\frac{dv_x}{d\xi}=-\frac{\gamma^2 n_i}{\alpha M}\frac{d\psi}{d\xi}+\frac{M}{\alpha n_i}\frac{dn_i}{d\xi} \tag{C.15}$$

then integrate with boundary conditions

$$\xi\to 0, v_x=0, n_i=1, \psi=0$$

we obtain

$$\alpha v_x+\frac{\gamma^2}{M}\left[\frac{fe^{\alpha_c\psi}}{\alpha_c}+(1-f)\left(\frac{e^{\alpha_h\psi}}{\alpha_h}-\beta\left(\psi-\frac{1}{\alpha_h}\right)\frac{e^{\alpha_h\psi}}{\alpha_h}+\beta\left(\psi^2-\frac{2\psi}{\alpha_h}+\frac{2}{\alpha_h^2}\right)\frac{e^{\alpha_h\psi}}{\alpha_h}\right)\right]=-\frac{M}{n_i}+C$$

then

$$\frac{\gamma^2}{M}\left[\frac{f}{\alpha_c}+(1-f)\left(\frac{1}{\alpha_h}+\frac{\beta}{\alpha_h^2}+\frac{2\beta}{\alpha_h^3}\right)\right]+M=C$$

we have

$$v_x=\frac{1}{\alpha}\left[M\left(1-\frac{1}{n_i}\right)-\frac{\gamma^2}{M}\left(\frac{f}{\alpha_c}(e^{\alpha_c\psi}-1)+(1-f)\left(\frac{1}{\alpha_h}(e^{\alpha_h\psi}-1)+\beta\psi(\psi-1)\frac{e^{\alpha_h\psi}}{\alpha_h}\right.\right.\right.$$
$$\left.\left.\left.-\frac{\beta}{\alpha_h^2}(1-e^{\alpha_h\psi}+2\psi e^{\alpha_h\psi}+\frac{2\beta}{\alpha_h^3}(e^{\alpha_h\psi}-1)\right)\right)\right] \tag{C.16}$$

equation (C.15) into (C.10)

$$-\frac{M}{n_i}\left(-\frac{\gamma^2 n_i}{\alpha M}\frac{d\psi}{d\xi}+\frac{M}{\alpha n_i^2}\frac{dn_i}{d\xi}\right)=-\alpha\frac{d\psi}{d\xi}+Mv_y$$

$$\left(\frac{\gamma^2+\alpha^2}{\alpha M}\right)\frac{d\psi}{d\xi}-\frac{M}{\alpha n_i^3}\frac{dn_i}{d\xi}=v_y$$

since

$$\gamma^2+\alpha^2=\cos^2\theta+\sin^2\theta=1$$

we have

$$\frac{1}{\alpha M}\left(\frac{d\psi}{d\xi}-\frac{M^2}{n_i^3}\frac{dn_i}{d\xi}\right)=v_y$$

$$\frac{1}{\alpha M}\left[\frac{d}{d\xi}\left(\psi+\frac{M^2}{2n_i^2}\right)\right]=v_y \tag{C.17}$$

from equation (C.11)

$$(-M+\alpha v_x+\gamma v_z)\frac{dv_y}{d\xi}=-Mv_x$$

$$-\frac{M}{n_i}\frac{dv_y}{d\xi}=-Mv_x$$

$$\frac{dv_y}{d\xi}=n_iv_x \tag{C.18}$$

equation (C.16), (C.17) and (C.18) together

$$\frac{d}{d\xi}\left[\frac{d}{d\xi}\left(\psi+\frac{M^2}{2n_i^2}\right)\right]=M^2(n_i-1)-n_i\gamma^2\left[\frac{f}{\alpha_c}(e^{\alpha_c\psi}-1)+(1-f)\right.$$
$$\left.\left(\frac{1}{\alpha_h}(e^{\alpha_h\psi}-1)+\beta\psi(\psi-1)\frac{e^{\alpha_h\psi}}{\alpha_h}-\frac{\beta}{\alpha_h^2}(1-e^{\alpha_h\psi}+2\psi e^{\alpha_h\psi})+\frac{2\beta}{\alpha_h^3}(e^{\alpha_h\psi}-1)\right)\right] \tag{C.19}$$

let

$$t=\psi+\frac{M^2}{2n_i^2}$$

then

$$\frac{dt}{d\xi}=\frac{d\psi}{d\xi}-\frac{M^2}{n_i^3}\frac{dn_i}{d\xi}$$

$$n_i=fe^{\alpha_c\psi}+(1-f)(1-\beta\psi+\beta\psi^2)e^{\alpha_h\psi}$$

$$\frac{dn_i}{d\xi}=\left[f\alpha_ce^{\alpha_c\psi}+(1-f)\left(\alpha_he^{\alpha_h\psi}-\beta(1+\alpha_h\psi)e^{\alpha_h\psi}+\beta\psi(2+\alpha_h\psi)e^{\alpha_h\psi}\right)\right]\frac{d\psi}{d\xi}$$

and

$$\frac{dt}{d\xi}=\frac{d\psi}{d\xi}-\frac{M^2}{n_i^3}\left[f\alpha_ce^{\alpha_c\psi}+(1-f)\left(\alpha_he^{\alpha_h\psi}-\beta(1+\alpha_h\psi)e^{\alpha_h\psi}+\beta\psi(2+\alpha_h\psi)e^{\alpha_h\psi}\right)\right]\frac{d\psi}{d\xi}$$

now, multiply both side of equation (C.19) with $2\frac{dt}{d\xi}$ and integrate, we obtain

$$\frac{1}{2}\left(\frac{dt}{d\xi}\right)^2 = -\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\gamma^2)\psi + M^2P(\psi) - \frac{\gamma^2 P(\psi)^2}{2} - \frac{\gamma^2 M^2 P(\psi)}{n_i}$$

then, we obtain

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + \frac{1}{\left(1 - \frac{M^2(f\alpha_c\exp(\alpha_c\psi)+(1-f)(\alpha_h\exp(\alpha_h\psi)-\beta(1+\alpha_h\psi)\exp(\alpha_h\psi)+\beta\psi(2+\alpha_h\psi)\exp(\alpha_h\psi)))}{(f\exp(\alpha_c\psi)+(1-f)(1-\beta\psi+\beta\psi^2)\exp(\alpha_h\psi))^3}\right)^2}\times$$
$$-\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\gamma^2)\psi + M^2P(\psi) - \frac{\gamma^2 P(\psi)^2}{2} - \frac{\gamma^2 M^2 P(\psi)}{n_i} = 0 \qquad \text{(C.20)}$$

where

$$P(\psi) = \frac{f}{\alpha_c}(\exp(\alpha_c\psi)-1) + \frac{(1-f)}{\alpha_h}\big((\exp(\alpha_h\psi)-1)$$
$$+\beta\psi(\psi-1)\exp(\alpha_h\psi) - \frac{\beta}{\alpha_h}(1-\exp(\alpha_h\psi)+2\psi\exp(\alpha_h\psi))$$
$$+\frac{2\beta}{\alpha_h^2}(\exp(\alpha_h\psi)-1)\Big) \qquad \text{(C.21)}$$

Appendix D

Algebraic expression for the Sagdeev potential in a magnetized plasma with two ion species and electrons

The density and temperature of the Boltzmann distributed cool electrons (n_e, T_e) and hot protons (n_p, T_p) in normalized form are given by

cool electron

$$n_e = n_{eo} \exp\left(\frac{e\phi}{T_e}\right) \tag{D.1}$$

hot proton

$$n_p = n_{po} \exp\left(\frac{-e\phi}{T_p}\right) \tag{D.2}$$

and

$$n_e = n_i + n_p \tag{D.3}$$

now, let $\psi = \frac{e\phi}{T_e}$, $\frac{n_{po}}{n_{eo}} = p$

$$n_e = \frac{n_{eo}}{n_{eo}} \exp(\psi) = \exp(\psi) \tag{D.4}$$

$$\begin{aligned} n_p &= \frac{n_{po}}{n_{eo}} \exp\left(\frac{-e\phi}{T_p}\right) \\ &= g \exp\left(\frac{-e\phi}{T_e}\frac{T_e}{T_p}\right) \end{aligned}$$

$$n_p = g\exp(-\alpha_T\psi) \tag{D.5}$$

where $\alpha_T = \frac{T_e}{T_p}$

Therefore,

$$n_i = \frac{\exp(\psi) - g\exp(-\alpha_T\psi)}{1-g} \tag{D.6}$$

Magnetized cold oxygen ion beam (described by the fluid equations) (normalized)

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_{ix})}{\partial x} + \frac{\partial(n_i v_{iz})}{\partial z} = 0 \tag{D.7}$$

$$\frac{\partial v_{ix}}{\partial t} + \left(v_{ix}\frac{\partial}{\partial x} + v_{iz}\frac{\partial}{\partial z}\right) v_{ix} = -\frac{e}{m_i}\frac{\partial\phi}{\partial x} + \Omega_i v_{iy} \tag{D.8}$$

$$\frac{\partial v_{iy}}{\partial t} + \left(v_{ix}\frac{\partial}{\partial x} + v_{iz}\frac{\partial}{\partial z}\right) v_{iy} = -\Omega_i v_{ix} \tag{D.9}$$

$$\frac{\partial v_{iz}}{\partial t} + \left(v_{ix}\frac{\partial}{\partial x} + v_{iz}\frac{\partial}{\partial z}\right) v_{iz} = -\frac{e}{m_i}\frac{\partial\phi}{\partial z} \tag{D.10}$$

dimensionless variables: $\iota = \Omega_i t$, $(\eta, \zeta) = (x, z)/\rho_s$, $V_k = v_{ik}/c_s$ (where $k = x, z$), $\Omega = c_s/\rho_s$. Then, equation (D.7)-(D.10) becomes

$$\frac{\partial n_i}{\partial \iota} + \frac{\partial(n_i v_x)}{\partial \eta} + \frac{\partial(n_i v_z)}{\partial \zeta} = 0 \tag{D.11}$$

$$\frac{\partial v_x}{\partial \iota} + \left(v_x\frac{\partial}{\partial \eta} + v_z\frac{\partial}{\partial \zeta}\right) v_x = -\frac{\partial\psi}{\partial \eta} + v_y \tag{D.12}$$

$$\frac{\partial v_y}{\partial \iota} + \left(v_x\frac{\partial}{\partial \eta} + v_z\frac{\partial}{\partial \zeta}\right) v_y = -v_x \tag{D.13}$$

$$\frac{\partial v_z}{\partial \iota} + \left(v_x\frac{\partial}{\partial \eta} + v_z\frac{\partial}{\partial \zeta}\right) v_z = -\frac{\partial\psi}{\partial \zeta} \tag{D.14}$$

define a localized stationary frame

$$\xi = (\alpha\eta + \gamma\zeta - M\iota)/M \tag{D.15}$$

where

$$\partial_\iota = -\partial_\xi, \partial_\eta = \frac{\alpha}{M}\partial_\xi, \partial_\zeta = \frac{\gamma}{M}\partial_\xi \tag{D.16}$$

from equation (D.11)

$$-\frac{dn_i}{d\xi} + \frac{\alpha}{M}\frac{dn_i v_x}{d\xi} + \frac{\gamma}{M}\frac{dn_i v_x}{d\xi} = 0$$

$$M\frac{dn_i}{d\xi} = \frac{dn_i}{d\xi}(\alpha v_x + \gamma v_z)$$

integrate with the boundary condition

$$\xi \to 0, n_i \to 1, \psi = 0, v_x = 0, v_z = v_o$$

we have

$$Mn_i + C = n_i(\alpha v_x + \gamma v_z)$$

$$M + C = \gamma v_o$$

$$C = \delta - M \tag{D.17}$$

where $\delta = \gamma v_o$

$$n_i(\alpha v_x + \gamma v_z) = Mn_i + \delta - M$$

$$\alpha v_x + \gamma v_z = M - \frac{M - \delta}{n_i} \tag{D.18}$$

from equation (D.12)

$$-\frac{dv_x}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_x = -\frac{\alpha}{M}\frac{d\psi}{d\xi} + v_y$$

$$-M\frac{dv_x}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y$$

$$(-M + \alpha v_x + \gamma v_z)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y$$

$$\left(-M + M - \frac{M - \delta}{n_i}\right)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y$$

$$-\left(\frac{M - \delta}{n_i}\right)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y \tag{D.19}$$

from equation (D.13)

$$-\frac{dv_y}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_y = -v_x$$

$$-M\frac{dv_y}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_y}{d\xi} = -Mv_x$$

$$\left(-M + M - \frac{M - \delta}{n_i}\right)\frac{dv_y}{d\xi} = -Mv_x$$

$$\left(\frac{M-\delta}{n_i}\right)\frac{dv_y}{d\xi} = Mv_x \tag{D.20}$$

from equation (D.14)

$$-\frac{dv_z}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_z = -\frac{\gamma}{M}\frac{d\psi}{d\xi}$$

$$-M\frac{dv_z}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}$$

$$\left(-M + M - \frac{M-\delta}{n_i}\right)\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}$$

$$\left(\frac{M-\delta}{n_i}\right)\frac{dv_z}{d\xi} = \gamma\frac{d\psi}{d\xi} \tag{D.21}$$

now, differentiate equation (D.18) w. r .t. $d\Omega$, we have

$$\alpha\frac{dv_x}{d\xi} + \gamma\frac{dv_z}{d\xi} = \left(\frac{M-\delta}{n_i^2}\right)\frac{dn_i}{d\xi} \tag{D.22}$$

substitute equation (D.21) in equation (D.22)

$$\frac{dv_x}{d\xi} = -\left(\frac{\gamma^2 n_i}{\alpha(M-\delta)}\right)\frac{d\psi}{d\xi} + \left(\frac{M-\delta}{\alpha n_i^2}\right)\frac{dn_i}{d\xi} \tag{D.23}$$

integrate equation (D.23), we have

$$\alpha v_x + \frac{\gamma^2}{M-\delta}\left[\frac{1}{1-g}\left(e^x + \frac{g}{\alpha_T}e^{-\alpha_T\psi}\right)\right] = -\frac{M-\delta}{n_i} + C \tag{D.24}$$

using boundary conditions

$$v_x = 0, n_i = 1, \psi = 0$$

we obtain

$$\frac{\gamma^2}{M-\delta}\left[\frac{1}{1-g}\left(1 + \frac{g}{\alpha_T}\right)\right] + (M-\delta) = C$$

$$\alpha v_x = (M-\delta)\left(1 - \frac{1}{n_i}\right) - \frac{\gamma^2}{M-\delta}\left[\frac{1}{1-g}\left((e^\psi - 1) + \frac{g}{\alpha_T}(e^{-\alpha_T\psi} - 1)\right)\right] \tag{D.25}$$

equation (D.23) into (D.19), we have

$$\frac{\gamma^2}{\alpha}\frac{d\psi}{d\xi} - \frac{(M-\delta)^2}{\alpha n_i^3}\frac{dn_i}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y$$

$$\frac{1}{\alpha M}\left[\frac{d\psi}{d\xi} - \frac{(M-\delta)^2}{n_i^3}\frac{dn_i}{d\xi}\right] = v_y \tag{D.26}$$

since

$$\alpha^2 + \gamma^2 = 1$$

equation (D.20) into (D.26), we have

$$\frac{1}{\alpha M}\left[\frac{d}{d\xi}\left(\frac{d\psi}{d\xi} - \frac{(M-\delta)^2}{n_i^3}\frac{dn_i}{d\xi}\right)\right] = \frac{n_i M v_x}{M-\delta}$$

$$\frac{d}{d\xi}\left[\frac{d}{d\xi}\left(\psi + \frac{(M-\delta)^2}{2n_i^2}\right)\right] = \frac{\alpha n_i M^2 v_x}{M-\delta}$$

$$\frac{d}{d\xi}\left[\frac{d}{d\xi}\left(\psi + \frac{(M-\delta)^2}{2n_i^2}\right)\right] = M^2(n_i-1) - \frac{\gamma^2 M^2 n_i}{(M-\delta)^2}\left[\frac{1}{1-g}\left((e^{\psi}-1) + \frac{g}{\alpha_T}(e^{-\alpha_T\psi}-1)\right)\right] \tag{D.27}$$

let

$$\chi = \psi + \frac{(M-\delta)^2}{2n_i^2}$$

$$\frac{d\chi}{d\xi} = \frac{d\psi}{d\xi} - \frac{(M-\delta)^2}{n_i^3}\frac{dn_i}{d\xi}$$

or

$$\frac{d\chi}{d\xi} = \left(1 - \frac{(M-\delta)^2\left(\frac{1}{1-g}(e^{\psi} + \alpha_T g e^{-\alpha_T\psi}\right)}{n_i^3}\right)\frac{d\psi}{d\xi} \tag{D.28}$$

Multiply both sides of equation (D.27) by $2\frac{d\chi}{d\xi}$ and integrate

$$\begin{aligned}
\frac{1}{2}\left(\frac{d\chi}{d\xi}\right)^2 &= \Bigg(-\frac{M^2(M-\delta)^2}{2n_i^2}(1-n_i)^2 \\
&-M^2(1-\gamma^2)\psi + M^2\left(\frac{1}{1-g}\left((\exp(\psi)-1) + \frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right) \\
&-\frac{\gamma^2}{2(M-\delta)^2}\left(\frac{1}{1-g}\left((\exp(\psi)-1) + \frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)^2 \\
&-\frac{M^2\gamma^2}{n_i}\left(\frac{1}{1-g}\left((\exp(\psi)-1) + \frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)\Bigg)
\end{aligned} \tag{D.29}$$

equation (D.28) into (D.29), we have

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 = \frac{1}{\left(1-\frac{(M-\delta)^2}{n_i^3}\left(\frac{1}{1-g}(\exp(\psi)+g\alpha_T\exp(-\alpha_T\psi)\right)\right)^2}\times\left(-\frac{M^2(M-\delta)^2}{2n_i^2}(1-n_i)^2\right.$$
$$-M^2(1-\gamma^2)\psi+M^2\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)$$
$$-\frac{\gamma^2}{2(M-\delta)^2}\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)^2$$
$$\left.-\frac{M^2\gamma^2}{n_i}\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)\right)=0 \tag{D.30}$$

which can be written as "Energy integral equation"

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2+V(\psi,M)=0 \tag{D.31}$$

where

$$V(\psi,M) = -\frac{1}{\left(1-\frac{(M-\delta)^2}{n_i^3}\left(\frac{1}{1-g}(\exp(\psi)+g\alpha_T\exp(-\alpha_T\psi)\right)\right)^2}\times\left(-\frac{M^2(M-\delta)^2}{2n_i^2}(1-n_i)^2\right.$$
$$-M^2(1-\gamma^2)\psi+M^2\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)$$
$$-\frac{\gamma^2}{2(M-\delta)^2}\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)^2$$
$$\left.-\frac{M^2\gamma^2}{n_i}\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)\right) \tag{D.32}$$

at beam velocity = 0 (i.e., $\delta=\gamma v_o=0$).

$$V(\psi,M) = -\frac{1}{\left(1-\frac{M^2}{n_i^3}\left(\frac{1}{1-g}(\exp(\psi)+g\alpha_T\exp(-\alpha_T\psi)\right)\right)^2}\times\left(-\frac{M^4}{2n_i^2}(1-n_i)^2\right.$$
$$-M^2(1-\gamma^2)\psi+M^2\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)$$
$$-\frac{\gamma^2}{2}\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)^2$$
$$\left.-\frac{M^2\gamma^2}{n_i}\left(\frac{1}{1-g}\left((\exp(\psi)-1)+\frac{g}{\alpha_T}(\exp(-\alpha_T\psi)-1)\right)\right)\right) \tag{D.33}$$

Appendix E

Algebraic expression for the Sagdeev potential in a magnetized plasma with cold oxygen ions, cool ions and two temperature electrons

The density and temperature of the Boltzmann distribution of the cool (n_{ce}, T_{ce}) and hot (n_{he}, T_{he}) electrons and cool (n_{ci}, T_{ci}) ion species and are given in normalized form as follows:

cool electrons:

$$n_{ce} = n_{ce0} \exp\left(\frac{e\phi}{T_{ce}}\right)$$

$$n_{ce} = \frac{n_{ce0}}{n_{i0}} \exp\left(\frac{e\phi}{T_{eff}} \frac{T_{eff}}{T_{ce}}\right)$$

$$n_{ce} = f \exp(\alpha_{ce}\psi) \tag{E.1}$$

hot electrons:

$$n_{he} = n_{he0} \exp\left(\frac{e\phi}{T_{he}}\right)$$

$$n_{he} = \frac{n_{he0}}{n_{i0}} \exp\left(\frac{e\phi}{T_{eff}} \frac{T_{eff}}{T_{he}}\right)$$

$$n_{he} = (1-f) \exp(\alpha_{he}\psi) \tag{E.2}$$

Cool ions:

$$n_{ci} = n_{ci0}\exp\left(-\frac{e\phi}{T_{ci}}\right)$$

$$n_{ci} = \frac{n_{ci0}}{n_{i0}}\exp\left(-\frac{e\phi}{T_{eff}}\frac{T_{eff}}{T_{ci}}\right)$$

$$n_{ci} = g\exp(-\alpha_{ci}\psi) \tag{E.3}$$

Magnetized Cool Oxygen ions (normalized)(described by the fluid equations) :

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_x)}{\partial x} + \frac{\partial (n_i v_z)}{\partial z} = 0 \tag{E.4}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial \psi}{\partial x} + v_y \tag{E.5}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_y = -v_x \tag{E.6}$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial \psi}{\partial z} \tag{E.7}$$

The quasi-neutrality condition

$$n_i = n_{ce} + n_{he} - n_{ci} = \frac{fe^{\alpha_{ce}\psi} + (1-f)e^{\alpha_{he}\psi} - ge^{-\alpha_{ci}\psi}}{1-g} \tag{E.8}$$

stationary frame

$$\xi = (\alpha x + \gamma z - Mt)/M \tag{E.9}$$

$$\frac{\partial \xi}{\partial x} = \frac{\alpha}{M}, \frac{\partial \xi}{\partial z} = \frac{\gamma}{M}, \frac{\partial \xi}{\partial t} = -1$$

from equation (E.4)

$$-\frac{dn_i}{d\xi} + \frac{\alpha}{M}\frac{dn_i v_x}{d\xi} + \frac{\gamma}{M}\frac{dn_i v_x}{d\xi} = 0$$

$$M\frac{dn_i}{d\xi} = \frac{dn_i}{d\xi}(\alpha v_x + \gamma v_z)$$

integrate with the boundary condition

$$\xi \to 0, n_i \to 1, \psi = 0, v_x = v_z = 0$$

then

$$M = -C$$

$$Mn_i - M = n_i(\alpha v_x + \gamma v_z)$$

$$\alpha v_x + \gamma v_z = M\left(1 - \frac{1}{n_i}\right) \tag{E.10}$$

from equation (E.5)

$$-\frac{dv_x}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_x = -\frac{\alpha}{M}\frac{d\psi}{d\xi} + v_y$$

$$-M\frac{dv_x}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y$$

$$\left(-M + M - \frac{M}{n_i}\right)\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y$$

$$-\frac{M}{n_i}\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y \tag{E.11}$$

from equation (E.6)

$$-\frac{dv_y}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_y = -v_x$$

$$-M\frac{dv_y}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_y}{d\xi} = -Mv_x$$

$$\left(-M + M - \frac{M}{n_i}\right)\frac{dv_y}{d\xi} = -Mv_x$$

$$\frac{1}{n_i}\frac{dv_y}{d\xi} = v_x \tag{E.12}$$

from equation (E.7)

$$-\frac{dv_z}{d\xi} + \left(\frac{\alpha}{M}v_x\frac{d}{d\xi} + \frac{\gamma}{M}v_z\frac{d}{d\xi}\right)v_z = -\frac{\gamma}{M}\frac{d\psi}{d\xi}$$

$$-M\frac{dv_z}{d\xi} + (\alpha v_x + \gamma v_z)\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}$$

$$\left(-M + M - \frac{M}{n_i}\right)\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}$$

$$\frac{M}{n_i}\frac{dv_z}{d\xi} = \gamma\frac{d\psi}{d\xi} \tag{E.13}$$

differentiate equation (E.10) w.r.t . ∂_ξ, we have

$$\alpha\frac{dv_x}{d\xi}+\gamma\frac{dv_z}{d\xi}=\frac{M}{n_i^2}\frac{dn_i}{d\xi} \tag{E.14}$$

equation (E.13) into (E.14)

$$\alpha\frac{dv_x}{d\xi}+\gamma^2\left(\frac{n_i}{M}\frac{d\psi}{d\xi}\right)=\frac{M}{n_i^2}\frac{dn_i}{d\xi}$$

i.e

$$\frac{dv_x}{d\xi}=-\frac{\gamma^2 n_i}{\alpha M}\frac{d\psi}{d\xi}+\frac{M}{\alpha n_i^2}\frac{dn_i}{d\xi} \tag{E.15}$$

integrate equation(E.15)

$$\alpha\int\frac{dv_x}{d\xi}d\xi+\frac{\gamma^2}{M}\int n_i\frac{d\psi}{d\xi}d\xi=M\int\frac{1}{n_i^2}\frac{dn_i}{d\xi}d\xi$$

$$\alpha v_x+\frac{\gamma^2}{M}\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}e^{\alpha_{ce}\psi}+\frac{1-f}{\alpha_{he}}e^{\alpha_{he}\psi}+\frac{g}{\alpha_{ci}}e^{-\alpha_{ci}\psi}\right)\right)=-\frac{M}{n_i}+C$$

using boundary conditions:

$$v_x=0, n_i=1, \psi=0$$

we have

$$\frac{\gamma^2}{M}\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}+\frac{1-f}{\alpha_{he}}+\frac{g}{\alpha_{ci}}\right)\right)+M=C$$

therefore,

$$\alpha v_x=M\left(1-\frac{1}{n_i}\right)-\frac{\gamma^2}{M}\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi}-1)+\frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi}-1)+\frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi}-1)\right)\right) \tag{E.16}$$

equation (E.15) into (E.11)

$$\frac{\gamma^2}{\alpha}\frac{d\psi}{d\xi}-\frac{M^2}{\alpha n_i^3}\frac{dn_i}{d\xi}=-\alpha\frac{d\psi}{d\xi}+Mv_y$$

$$\left(\frac{\gamma^2+\alpha^2}{\alpha}\right)\frac{d\psi}{d\xi}-\frac{M^2}{\alpha n_i^3}\frac{dn_i}{d\xi}=Mv_y$$

Since

$$\alpha^2+\gamma^2=1$$

$$\frac{1}{\alpha M}\left(\frac{d\psi}{d\xi}-\frac{M^2}{n_i^3}\frac{dn_i}{d\xi}\right)=v_y \tag{E.17}$$

equation (E.17) into (E.12)

$$\frac{d}{d\xi}\left(\frac{d\psi}{d\xi} - \frac{M^2}{n_i^3}\frac{dn_i}{d\xi}\right) = \alpha M n_i v_x \tag{E.18}$$

equation (E.18) becomes

$$\frac{d}{d\xi}\left(\frac{d}{d\xi}\left(\psi + \frac{M^2}{2n_i^2}\right)\right) = M^2(n_i - 1) - \gamma^2 n_i \left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi} - 1)\right.\right.$$
$$\left.\left. + \frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi} - 1) + \frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi} - 1)\right)\right) \tag{E.19}$$

let

$$t = \psi + \frac{M^2}{2n_i^2}$$

$$\frac{dt}{d\xi} = \frac{d\psi}{d\xi} - \frac{M^2}{n_i^3}\frac{dn_i}{d\xi}$$

$$\frac{dn_i}{d\xi} = \frac{1}{1-g}\left(f\alpha_{ce}e^{\alpha_{ce}\psi} + (1-f)\alpha_{he}e^{\alpha_{he}\psi} + g\alpha_{ci}e^{-\alpha_{ci}\psi}\right)\frac{d\psi}{d\xi}$$

i.e

$$\frac{dt}{d\xi} = \left(1 - \frac{M^2}{n_i^3}\left(\frac{1}{1-g}\left(f\alpha_{ce}e^{\alpha_{ce}\psi} + (1-f)\alpha_{he}e^{\alpha_{he}\psi} + g\alpha_{ci}e^{-\alpha_{ci}\psi}\right)\right)\right)\frac{d\psi}{d\xi}$$

Multiply both side of equation (E.19) by $2\frac{dt}{d\xi}$ and integate

$$\int 2\frac{dt}{d\xi}\frac{d}{d\xi}\left(\frac{dt}{d\xi}\right)d\xi = 2\left[\int M^2(n_i - 1) - \gamma^2 n_i \left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi} - 1)\right.\right.\right.$$
$$\left.\left.\left. + \frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi} - 1) + \frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi} - 1)\right)\right)\right]\frac{dt}{d\xi}$$

we obtain

$$\frac{1}{2}\left(\frac{dt}{d\xi}\right)^2 = \Bigg(-\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\gamma^2)\psi$$
$$+M^2\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi}-1)+\frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi}-1)+\frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi}-1)\right)\right)$$
$$-\frac{\gamma^2}{2}\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi}-1)+\frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi}-1)+\frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi}-1)\right)\right)^2$$
$$-\frac{M^2\gamma^2}{n_i}\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi}-1)+\frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi}-1)+\frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi}-1)\right)\right)\Bigg)$$

then, substitute for t

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 = \frac{1}{\left(1-\frac{M^2}{n_i^3}\left(\frac{1}{1-g}(f\alpha_{ce}e^{\alpha_{ce}\psi}+(1-f)\alpha_{he}e^{\alpha_{he}\psi}+g\alpha_{ci}e^{-\alpha_{ci}\psi}\right)\right)^2}\times$$
$$\Bigg(-\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\gamma^2)\psi$$
$$+M^2\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi}-1)+\frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi}-1)+\frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi}-1)\right)\right)$$
$$-\frac{\gamma^2}{2}\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi}-1)+\frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi}-1)+\frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi}-1)\right)\right)^2$$
$$-\frac{M^2\gamma^2}{n_i}\left(\frac{1}{1-g}\left(\frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi}-1)+\frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi}-1)+\frac{g}{\alpha_{ci}}(e^{-\alpha_{ci}\psi}-1)\right)\right)\Bigg) \quad \text{(E.20)}$$

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